Chapter 13 Test Review

13-1

Representing Sample Spaces

Represent a Sample Space The sample space of an experiment is the set of all possible outcomes. A sample space can be found using an organized list, table, or tree diagram.

Example: Maurice packs suits, shirts, and ties that can be mixed and matched. Using the packing list at the right, draw a tree diagram to represent the sample space for business suit combinations.

The sample space is the result of three stages:
- Suit color (G, B, or K)
- Shirt color (W or L)
- Tie (T or NT)

Maurice's Packing List
1. Suits: Gray, black, khaki
2. Shirts: White, light blue
3. Ties: Striped (But optional)

![Tree Diagram]

Exercises

Represent the sample space for each experiment by making a tree diagram.

1. The baseball team can wear blue or white shirts with blue or white pants.

2. The dance club is going to see either Sleeping Beauty or The Nutcracker at either Symphony Hall or The Center for the Arts.

3. Mikey's baby sister can drink either apple juice or milk from a bottle or a toddler cup.

4. The first part of the test consisted of two true-or-false questions.
13-1 Study Guide and Intervention (continued)
Representing Sample Spaces

Fundamental Counting Principle The number of all possible outcomes for an experiment can be found by multiplying the number of possible outcomes from each stage or event.

Example: The pattern for a certain license plate is 3 letters followed by 3 numbers. The letter “O” is not used as any of the letters and the number “0” is not used as any of the numbers. Any other letter or number can be used multiple times. How many license plates can be created with this pattern?

Use the Fundamental Counting Principle.

<table>
<thead>
<tr>
<th>1st Space</th>
<th>2nd Space</th>
<th>3rd Space</th>
<th>4th Space</th>
<th>5th Space</th>
<th>6th Space</th>
<th>Possible Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25</td>
<td>25</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>= 11,390,625</td>
</tr>
</tbody>
</table>

So 11,390,625 license plates can be created with this pattern.

Exercises

Find the number of possible outcomes for each situation.

1. A room is decorated with one choice from each category.

<table>
<thead>
<tr>
<th>Bedroom Décor</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paint color</td>
<td>8</td>
</tr>
<tr>
<td>Comforter set</td>
<td>6</td>
</tr>
<tr>
<td>Sheet set</td>
<td>8</td>
</tr>
<tr>
<td>Throw rug</td>
<td>5</td>
</tr>
<tr>
<td>Lamp</td>
<td>3</td>
</tr>
<tr>
<td>Wall hanging</td>
<td>5</td>
</tr>
</tbody>
</table>

= 24,400

2. A lunch at Lincoln High School contains one choice from each category.

<table>
<thead>
<tr>
<th>Cafeteria Meal</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main dish</td>
<td>3</td>
</tr>
<tr>
<td>Side dish</td>
<td>4</td>
</tr>
<tr>
<td>Vegetable</td>
<td>2</td>
</tr>
<tr>
<td>Salad</td>
<td>2</td>
</tr>
<tr>
<td>Salad Dressing</td>
<td>3</td>
</tr>
<tr>
<td>Dessert</td>
<td>2</td>
</tr>
<tr>
<td>Drink</td>
<td>3</td>
</tr>
</tbody>
</table>

= 824

3. In a catalog of outdoor patio plans, there are 4 types of stone, 3 types of edgers, 5 dining sets and 6 grills. Carl plans to order one item from each category.

= 360

4. The drama club held tryouts for 6 roles in a one-act play. Five people auditioned for lead female, 3 for lead male, 8 for the best friend, 4 for the mom, 2 for the dad, and 3 for the crazy aunt.

= 2,880
13-2 Study Guide and Intervention

Probability with Permutations and Combinations

Probability Using Permutations A permutation is an arrangement of objects where order is important. To find the number of permutations of a group of objects, use the factorial. A factorial is written using a number and !.

The following are permutation formulas:

<table>
<thead>
<tr>
<th>n distinct objects taken r at a time</th>
<th>[ nP_r = \frac{n!}{(n-r)!} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>n objects, where one object is repeated r₁ times, another is repeated r₂ times, and so on</td>
<td>[ \frac{n!}{r₁! \cdot r₂! \cdot \ldots \cdot r_k!} ]</td>
</tr>
<tr>
<td>n objects arranged in a circle with no fixed reference point</td>
<td>[ \frac{n!}{n} \text{ or } (n-1)! ]</td>
</tr>
</tbody>
</table>

Example: The cheer squad is made up of 12 girls. A captain and a co-captain are selected at random. What is the probability that Chantel and Cadence are chosen as leaders?

Find the number of possible outcomes.

\[ 12P_2 = \frac{12!}{(12-2)!} = \frac{12!}{10!} = 12 \cdot 11 = 132 \]

Find the number of favorable outcomes.

\[ 2! = 2 \]

The probability of Chantel and Cadence being chosen is

\[ \frac{\text{favorable outcomes}}{\text{total number of outcomes}} = \frac{2}{132} = \frac{1}{66} \]

Exercises

1. BOOKS You have a textbook for each of the following subjects: Spanish, English, Chemistry, Geometry, History, and Psychology. If you choose 4 of these at random to arrange on a shelf, what is the probability that the Geometry textbook will be first from the left and the Chemistry textbook will be second from the left?

\[ 6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 30 \]

2. CLUBS The Service Club is choosing members at random to attend one of four conferences in LA, Atlanta, Chicago, and New York. There are 20 members in the club. What is the probability that Lana, Sherry, Miguel, and Jerome are chosen for these trips?

\[ \binom{20}{4} = \frac{20!}{4! \cdot (20-4)!} = \frac{20!}{4! \cdot 16!} = 4845 \]

3. TELEPHONE NUMBERS What is the probability that a 7-digit telephone number generated using the digits 2, 3, 2, 5, 2, 7, and 3 is the number 222-3357?

\[ \binom{7-1}{2} = \binom{6}{2} = \frac{6!}{2! \cdot 4!} = 15 \cdot 2 = 30 \]

\[ \binom{7-1}{3} = \binom{6}{3} = \frac{6!}{3! \cdot 3!} = 20 \]

\[ \binom{7-1}{2} = \binom{6}{2} = \frac{6!}{2! \cdot 4!} = 15 \cdot 2 = 30 \]

\[ \binom{7-1}{3} = \binom{6}{3} = \frac{6!}{3! \cdot 3!} = 20 \]

4. DINING OUT A group of 4 girls and 4 boys is randomly seated at a round table. What is the probability that the arrangement is boy-girl-boy-girl?

\[ \text{SKIP} \]
Probability Using Combinations A combination is an arrangement of objects where order is NOT important. To find the number of combinations of \( n \) distinct objects taken \( r \) at a time, denoted by \( \binom{n}{r} \), use the formula:

\[
\binom{n}{r} = \frac{n!}{(n-r)! r!}
\]

Example: Taryn has 15 soccer trophies but she only has room to display 9 of them. If she chooses them at random, what is the probability that each of the trophies from the school invitational from the 1st through 9th grades will be chosen?

**Step 1** Since the order does not matter, the number of possible outcomes is

\[
\binom{15}{9} = \frac{15!}{(15-9)!9!} = 5005
\]

**Step 2** There is only one favorable outcome—the 9 specific trophies being chosen.

**Step 3** The probability that these 9 trophies are chosen is

\[
\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{5005}
\]

Exercises

1. **ICE CREAM** Kali has a choice of 20 flavors for her triple scoop cone. If she chooses the flavors at random, what is the probability that the 3 flavors she chooses will be vanilla, chocolate, and strawberry?

\[
\binom{20}{3} = 1140
\]

2. **PETS** Dani has a dog walking business serving 9 dogs. If she chooses 4 of the dogs at random to take an extra trip to the dog park, what is the probability that Fifi, Gordy, Spike and Fluffy are chosen?

\[
\binom{9}{4} = 126
\]

3. **CRITIQUE** A restaurant critic has 10 new restaurants to try. If he tries half of them this week, what is the probability that he will choose The Fish Shack, Carly’s Place, Chez Henri, Casa de Jorge, and Grillarious?

\[
\binom{10}{5} = 252
\]

4. **CHARITY** Emily is giving away part of her international doll collection to charity. She has 20 dolls, each from a different country. If she selects 10 of them at random, what is the probability she chooses the ones from Ecuador, Paraguay, Chile, France, Spain, Sweden, Switzerland, Germany, Greece, and Italy?

\[
\binom{20}{10} = 18,475\,6
\]

5. **ROLLER COASTERS** An amusement park has 12 roller coasters. Four are on the west side of the park, 4 are on the east side, and 4 are centrally located. The park’s Maintenance Department randomly chooses 4 roller coasters for upgrades each month. What is the probability that all 4 roller coasters on the west side are chosen in March?

\[
\binom{12}{4} = 495
\]
13-3 Study Guide and Intervention
Geometric Probability

Probability with Length  Probability that involves a geometric measure is called geometric probability. One type of measure is length.

Look at line segment $KL$.
If a point, $M$, is chosen at random on the line segment, then
$$P(M \text{ is on } KL) = \frac{KL}{RS}$$

Example: Point $X$ is chosen at random on $AD$. Find the probability that $X$ is on $AB$.

$$P(X \text{ is on } AB) = \frac{AB}{AD}$$
$$= \frac{8}{16}$$
$$= \frac{1}{2} \cdot 0.5, \text{ or } 50\%$$

Simplify.

Exercises
Point $M$ is chosen at random on $ZP$. Find the probability of each event.

1. $P(M \text{ is on } ZQ)$
$$\frac{2}{10} = \frac{1}{5} = 20\%$$

2. $P(M \text{ is on } QR)$
$$\frac{3}{10} = 30\%$$

3. $P(M \text{ is on } RP)$
$$\frac{5}{10} = 50\%$$

4. $P(M \text{ is on } QP)$
$$\frac{6}{10} = 60\%$$

5. TRAFFIC LIGHT  In a 5-minute traffic cycle, a traffic light is green for 2 minutes 27 seconds, yellow for 6 seconds, and red for 2 minutes 27 seconds. What is the probability that when you get to the light it is green?

$$2.27 + .06 + 2.27 = 4.6\text{ total time}$$
$$\frac{2.27}{4.6} = .4934 = 49.3\%$$

6. GASOLINE  Your mom's mini van has a 24 gallon tank. What is the probability that, when the engine is turned on, the needle on the gas gauge is pointing between $\frac{1}{4}$ and $\frac{1}{2}$ full?

$$\frac{6}{24} = \frac{1}{4} = 25\%$$
Probability with Area Geometric probabilities can also involve area. When determining geometric probability with targets, assume that the object lands within the target area and that it is equally likely that the object will land anywhere in the region.

Example: Suppose a coin is flipped into a reflection pond designed with colored tiles that form 3 concentric circles on the bottom. The diameter of the center circle is 4 feet and the circles are spaced 2 feet apart. What is the probability the coin lands in the center?

\[ P(\text{coin lands in center}) = \frac{\text{area of center circle}}{\text{area of base of pond}} = \frac{4\pi}{36\pi} = \frac{1}{9}, \text{ about } 0.11, \text{ or } 11\% \]

Exercises

1. LANDING A parachutist needs to land in the center of a target on a rectangular field that is 120 yards by 30 yards. The target is a circular design with a 10 yard radius. What is the probability the parachutist lands somewhere in the target?

2. CLOCKS Jonas watches the second hand on an analog clock as it moves past the numbers. What is the probability that at any given time the second hand on a clock is between the 2- and the 3-hour numbers?

Find the probability that a point chosen at random lies in the shaded region.

3. Use the spinner to find each probability. If the spinner lands on a line it is spun again.

6. \[ P(\text{pointer landing on red}) = \frac{40}{360} = 11.1\% \]

7. \[ P(\text{pointer landing on blue}) = \frac{30}{360} = 8.3\% \]

8. \[ P(\text{pointer landing on green}) = \frac{50}{360} = 13.9\% \]
13-5 Study Guide and Intervention
Probabilities of Independent and Dependent Events

Independent and Dependent Events Compound events, or two or more simple events happening together, can be independent or dependant. Events are independent events if the probability of one event does not affect the probability of the other. Events are dependent events if one event in some way changes the probability that the other occurs. The following are the Multiplication Rules for Probability.

<table>
<thead>
<tr>
<th>Probability of Two Independent Events</th>
<th>( P(A \text{ and } B) = P(A) \cdot P(B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of Two Dependent Events</td>
<td>( P(A \text{ and } B) = P(A) \cdot P(B</td>
</tr>
</tbody>
</table>

\( P(B | A) \) is the conditional probability and is read the probability that event \( B \) occurs given that event \( A \) has already occurred.

Example: The P.E. teacher puts 10 red and 8 blue marbles in a bag. If a student draws a red marble, the student plays basketball. If a student draws a blue marble, the student practices long jump. Suppose Josh draws a marble, and not liking the outcome, he puts it back and draws a second time. What is the probability that on each draw his marble is blue?

Let \( B \) represent a blue marble.

\[
P(B \text{ and } B) = P(B) \cdot P(B) = \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}
\]

Probability of independent events

\[
P(B) = \frac{4}{9}
\]

So, the probability of Josh drawing two blue marbles is \( \frac{16}{81} \) or about 20%.

Exercises

Determine whether the events are independent or dependent. Then find the probability.

1. A king is drawn from a deck of 52 cards, then a coin is tossed and lands heads up.

\[
\frac{4}{52} \cdot \frac{1}{2} = \frac{4}{104} = 0.0384 = 3.8\%
\]

2. A spinner with 4 equally spaced sections numbered 1 through 4 is spun and lands on 1, then a die is tossed and rolls a 1.

\[
\frac{1}{4} \cdot \frac{1}{6} = \frac{1}{24} = 0.0416 = 4.2\%
\]

3. A red marble is drawn from a bag of 2 blue and 5 red marbles and not replaced, then a second red marble is drawn.

\[
\frac{5}{7} \cdot \frac{4}{6} = \text{one less red} \cdot \frac{5}{7} \cdot \frac{4}{6} = \text{one less marble in bag} = \frac{20}{42} = \frac{47161}{47161}
\]

4. A red marble is drawn from a bag of 2 blue and 5 red marbles and then replaced, then a red marble is drawn again.

\[
\frac{5}{7} \cdot \frac{5}{7} = \frac{25}{49} = 0.5102 = 51\%
\]
### 13-5 Study Guide and Intervention (continued)

**Probabilities of Independent and Dependent Events**

**Conditional Probabilities** Conditional probability is used to find the probability of dependent events. It also can be used when additional information is known about an event.

The conditional probability of $B$ given $A$ is $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$

where $P(A) \neq 0$.

**Example:** The Spanish Club is having a C in code Mayo fiesta. The 10 students randomly draw cards numbered with consecutive integers from 1 to 10. Students who draw odd numbers will bring main dishes. Students who draw even numbers will bring desserts. If Cynthia is bringing a dessert, what is the probability that she drew the number 10?

Since Cynthia is bringing dessert, she must have drawn an even number.

Let $A$ be the event that an even number is drawn.

Let $B$ be the event that the number 10 is drawn.

\[
P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}
\]

Conditional Probability

\[
= \frac{0.5 \cdot 0.1}{0.5} = 0.1
\]

$P(A) = \frac{1}{2} = 0.5$ and $P(B) = \frac{1}{10} = 0.1$

Simplify.

The probability Cynthia drew the number 10 is 0.1 or 10%.

### Exercises

1. A blue marble is selected at random from a bag of 3 red and 9 blue marbles and not replaced. What is the probability that a second marble selected will be blue?

\[
\frac{8}{11} = .7272 = 72.7%\]

2. A die is rolled. If the number rolled is less than 5, what is the probability that it is the number 2?

\[
\frac{1}{4} = 25%.
\]

3. A quadrilateral has a perimeter of 16 and all side lengths are even integers. What is the probability that the quadrilateral is a square?

\[
\frac{1}{4} = 25%.
\]

4. A spinner with 8 evenly sized sections and numbered 1 through 8 is spun. Find the probability that the number spun is 6 given that it is an even number.

\[
\frac{1}{4} = 25%.
\]
Mutually Exclusive Events If two events cannot happen at the same time, and therefore have no common outcomes, they are said to be mutually exclusive. The following are the Addition Rules for Probability:

| Probability of Mutually Exclusive Events | \( P(A \text{ or } B) = P(A) + P(B) \) |
| Probability of Non-Mutually Exclusive Events | \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \) |

Example: At the ballpark souvenir shop, there are 15 posters of the first baseman, 20 of the pitcher, 14 of the center fielder, and 12 of the shortstop. What is the probability that a fan choosing a poster at random will choose a poster of the center fielder or the shortstop?

These are mutually exclusive events because the posters are of two different players.
Let \( C \) represent selecting a poster of the center fielder.
Let \( S \) represent selecting a poster of the shortstop.
\[
P(C \text{ or } S) = P(C) + P(S) = \frac{14}{61} + \frac{12}{61} = \frac{26}{61} \text{ or about } 43\%
\]

Exercises

Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability. Round to the nearest hundredth.

1. SHELTER selecting a cat or dog at the animal shelter that has 15 cats, 25 dogs, 9 rabbits and 3 horses

   \[
   P(C \text{ or } D) = \frac{15}{52} + \frac{25}{52} = \frac{40}{52} = \frac{10}{13} \approx 0.7692 = 76.9\%
   \]

2. GAME rolling a 6 or an even number on a die while playing a game

   \[
   P(6 \text{ or } \text{even}) = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3} \approx 0.6667 = 66.7\%
   \]

3. AWARDS The student of the month gets to choose his or her award from 9 gift certificates to area restaurants, 8 CDs, 6 DVDs, or 5 gift cards to the mall. What is the probability that the student of the month chooses a CD or DVD?

   \[
   P(\text{CD or DVD}) = \frac{8}{54} + \frac{6}{54} = \frac{14}{54} = \frac{7}{27} \approx 0.2638 = 26.4\%
   \]

4. STUDENT COUNCIL According to the table shown at the right, what is the probability that a person on a student council committee is a junior or on the service committee?

<table>
<thead>
<tr>
<th>committee</th>
<th>Soph.</th>
<th>junior</th>
<th>senior</th>
</tr>
</thead>
<tbody>
<tr>
<td>service</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Advertising</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Dances</td>
<td>4</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Administrative Liaison</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

   Total = 46
13-6 Study Guide and Intervention (continued)  
Probabilities of Mutually Exclusive Events

Probabilities of Complements The complement of an event $A$ is all of the outcomes in the sample space that are not included as outcomes of event $A$.

<table>
<thead>
<tr>
<th>Probability of the Complement of an Event</th>
<th>$P(\text{not } A) = 1 - P(A)$</th>
</tr>
</thead>
</table>

Example: A school has a photography display of 100 pictures. One of the pictures will be chosen for display at the district office. Lorenzo has 3 pictures on display. What is the probability that one of his photographs is not chosen?

Let $A$ represent selecting one of Lorenzo’s photographs.
Then find the probability of the complement of $A$.

$$P(\text{not } A) = 1 - P(A)$$ Probability of a complement

$$= 1 - \frac{3}{100}$$ Substitution

$$= \frac{97}{100}$$ or 0.97 Simplify

The probability that one of Lorenzo’s photos is not selected is 97%.

Exercises

Determine the probability of each event.

1. If there is a 4 in 5 chance that your mom will tell you to clean your room today after school, what is the probability that she won’t?

$$1 - \frac{4}{5} = \frac{1}{5} = 20\%$$

2. What is the probability of drawing a card from a standard deck and not getting a spade?

$$1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4} = 75\%$$

3. What is the probability of flipping a coin and not landing on tails?

$$1 - \frac{1}{2} = \frac{1}{2} = 50\%$$

4. What is the probability of rolling a pair of dice and not rolling a 6?

$$1 - \frac{12}{36} = \frac{24}{36} = \frac{2}{3} = 66\%$$

5. A survey found that about 90% of the junior class is right handed. If 2 juniors are chosen at random out of 100 juniors, what is the probability that at least one of them is not right handed?

$$100 - 90\% = 10\%$$